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TM # 931139

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TECHNICAL MEMORANDUM

OUTPUT STATISTICS OF THE SALFAS FM NORMALIZER SMOOTHING/PREDICTION FUNCTION

Date: 2 April 1993

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ABSTRACT

An analytic performance evaluation methodology for the determination of the expected value and variance of the noise mean estimate of the smoothing/prediction function of the SALFAS FM normalizer is presented. These statistics are necessary for the determination of the output Receiver Operating Characteristic (ROC) curves of the normalizer.

REPORT DOCUMENTATION PAGE

FORM APPROVED
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2.. REPORT DATE
2 APR 1993

3. REPORT TYPE AND DATES COVERED
TECHNICAL MEMORANDUM

4. TITLE AND SUBTITLE
OUTPUT STATISTICS OF THE SALFAS FM NORMALIZER SMOOTHING/PREDICTION FUNCTION

5. FUNDING NUMBERS
B20020

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION
NUWC, CODE 3314

8. PERFORMING ORGANIZATION REPORT NUMBER
TM 931139

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Jeff Jones, NAVSEA (PEO-USWASTO)

10. SPONSORING
NUWC

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution Statement "A"
Approved for Public Release;
Distribution is unlimited

12b. DISTRIBUTION CODE
SEE ATTACHED

13. ABSTRACT (Maximum 200 words)
AN ANALYTIC PERFORMANCE EVALUATION METHODOLOGY FOR THE DETERMINATION OF THE EXPECTED VALUE AND VARIANCE OF THE NOISE MEAN ESTIMATE OF THE SMOOTHING/PREDICTION FUNCTION OF THE SALFAS FM NORMALIZER IS PRESENTED. THESE STATISTICS ARE NECESSARY FOR THE DETERMINATION OF THE OUTPUT RECEIVER OPERATING CHARACTERISTIC (ROC) CURVES OF THE NORMALIZER.

14. SUBJECT TERMS
SALFAS FM NORMALIZER, PREDICTION, SMOOTHING, EXPECTED VALUE, VARIANCE INDEPENDENT SAMPLES, BIAS

15. NUMBER OF PAGES
16

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	

ADMINISTRATIVE INFORMATION

This memorandum was prepared under NUWC Project Number B20030, as part of the Normalizer Task of the Active Processing Initiative. The Task Leader is Fyzodeen Khan; the Principal Investigator is Fyzodeen Khan, Code 3314, the NUWC program manager is Dave Ashworth, 33A and the program sponsor is Jeff Jones, NAVSEA (PEO-USWASTO).

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1.0 INTRODUCTION

The FM matched filter output envelope is normalized relative to the local noise background using an estimate of the noise mean level in each cell of interest. The normalizer algorithm is designed to provide robust and on-line adaptive processing features which include [1]:

a. Identification and removal from the noise estimation data window interfering target echoes, unwanted background clutter edges, and noise data statistical outliers which would contaminate the window and bias the noise mean estimate.

b. Estimation and compensation for underlying noise background dynamic (nonstationary) intensity variations from cell to cell such as may occur in reverberation-limited shallow water or convergence zone environments.

c. A sufficient number of effectively independent samples, thus achieving desired random error smoothing and minimizing normalizer SNR losses while maintaining desired control of false alarm rate.

Noise mean estimation is achieved in two steps as described in [1]. First the matched filter output envelopes are operated on by a sliding split-window, median-based, peak-shearing estimation process. This is designed to suppress interferences in the noise data estimation window. The second step uses a sliding split-window process with a forward/backward two pole exponential filter to produce an estimate of noise mean of the test cell located at the center of the window. The data recursive filter allows for enhanced following of arbitrary noise mean level fluctuations while maintaining desired random error reduction. [2]

Figures 1 and 2 shows the median based processing and the forward/backward smoothing and prediction structures respectively. The window and gap sizes in both processing functions can be different as shown in the figures. The analysis which follows determines the expected value and variance of the predicted noise mean estimates of the forward/backward processing as defined in [1] which are necessary for the generation of the normalizer output Receiver Operating Characteristic curves [2].

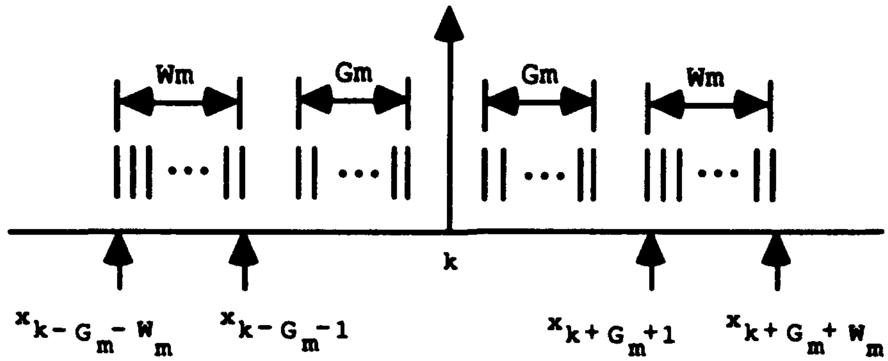

Figure 1 - Sliding split window structure for the median based, two-pass peak shearing function.

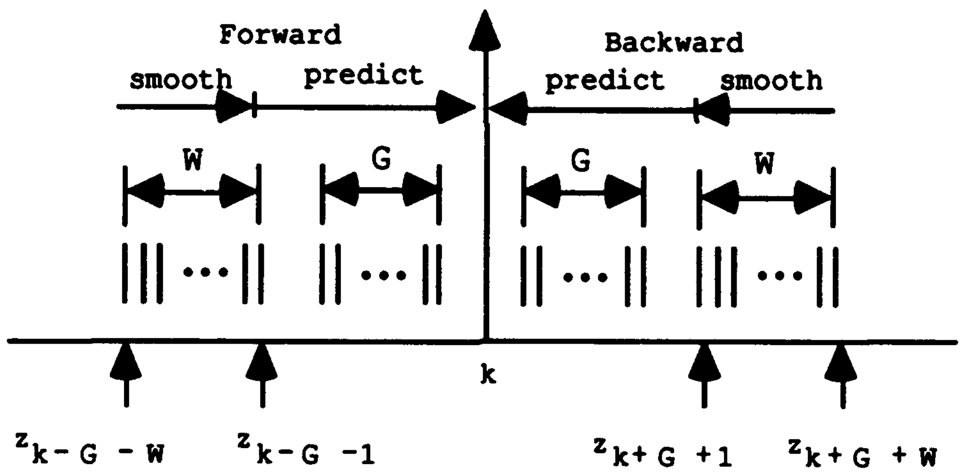

Figure 2 - Forward/backward smoothing and prediction function window structure.

2.0 FORWARD SMOOTHING AND PREDICTION

The output of the two pole recursive exponential smoothing filter in the forward direction is

$$sf1_i = sf1_{i-1} + \alpha\left[z_i - sf1_{i-1}\right] \tag{1a}$$

$$sf2_i = sf2_{i-1} + \alpha\left[sf1_i - sf2_{i-1}\right] \tag{1b}$$

where $sf1_i$ and $sf2_i$ are the outputs of filters 1 and 2 at time index i and z_i is the input to filter 1 (this is the output of the Median Processing Function).

In Vector-Matrix form the filter output is given by

$$\underline{sf}_i = A\,\underline{sf}_{i-1} + \underline{b}\,z_i \tag{2}$$

where

$$A = \begin{bmatrix} 1-\alpha & 0 \\ \alpha(1-\alpha) & 1-\alpha \end{bmatrix},$$

$$\underline{b}^T = \begin{bmatrix} \alpha & \alpha^2 \end{bmatrix},$$

$$\underline{sf}_i^T = \begin{bmatrix} sf1_i & sf2_i \end{bmatrix},$$

and $i = \{\, k-G-W+1, \ldots, k-G-1 \,\}$.

Equation (2) represents the output of the two pole recursive exponential filter at time i where i is the sample index in the forward smoothing window of size W, excluding the leftmost sample in the window at time $k-G-W$.

The forward predicted noise mean estimate of the test cell at time k in the middle of the normalizer window (see figure 2) is

$$\mu f_k = 2sf1_{k-G-1} - sf2_{k-G-1} + \frac{\alpha(G+1)}{1-\alpha}\left(sf1_{k-G-1} - sf2_{k-G-1}\right) \tag{3}$$

where μf_k is the predicted noise mean at time k and G is the normalizer half gap size. The predicted noise mean estimate μf_k is dependent on the output of the recursive filters at time $k-G-1$ and the constants α and G. Equation (3) can also be written in Vector-Matrix form as

$$\mu f_k = \underline{c}^T \, \underline{sf}_{k-G-1} \tag{4}$$

where $\underline{c}^T = [c_1 \quad c_2]$ and $c_1 = 2 + \dfrac{\alpha(G+1)}{1-\alpha}$, $c_2 = -\left(1 + \dfrac{\alpha(G+1)}{1-\alpha}\right)$.

Solving (2) iteratively with an initial state of $\underline{sf}_{k0} = \underline{1}\, z_{k0}$ the output of the forward smoothing filter is

$$\underline{sf}_i = A^{i-k0}\, \underline{1}\, z_{k0} + \sum_{\ell=k0+1}^{i} A^{i-\ell}\, \underline{b}\, z_\ell \tag{5}$$

where $\underline{1}^T = [1 \quad 1]$ and $k0 = k - G - W$.

The matrix form A^m in (5) represents an m-fold product $AxAx...xA$ $(A^0 = I)$ and is referred to as the *transition* or *fundamental* matrix of a system. Inspection of equation (5) reveals that the first term on the right represents the response of the system to the initial conditions only. The second term represents the response due to zero initial conditions and the input z and is essentially a convolution sum. Essential to the determination of the output is the m-fold matrix product A^m. A brute force method for determining A^m is undesirable since it would be time consuming for large m and insight into the structure of the filter response will not be obtained. Closed form solutions to the martrix polynomial A^m are obtainable [3, 4] and would be shown later.

The expected value of the forward smoothing function is

$$\mu_{\underline{sf}_i} = E\{\underline{sf}_i\} \tag{6}$$

$$= A^{i-k0}\, \underline{1}\, E\{z_{k0}\} + \sum_{\ell=k0+1}^{i} A^{i-\ell}\, \underline{b}\, E\{z_\ell\}$$

$$= A^{i-k0}\, \underline{1}\, \mu_{z_{k0}} + \sum_{\ell=k0+1}^{i} A^{i-\ell}\, \underline{b}\, \mu_{z_\ell}$$

where μ_{z_i} is the mean value of the median processed data at time sample i. Applying (4) the predicted mean in the forward direction at time k is

$$\mu f_k = \underline{c}^T \left[A^{W-1} \underline{1} z_{k0} + \sum_{\ell=k0+1}^{k-G-1} A^{k-G-1-\ell} \underline{b} z_\ell \right] \tag{7}$$

$$= \bar{\alpha}_0 z_{k0} + \sum_{m=0}^{W-2} \alpha_m z_{k-G-1-m}$$

where $\bar{\alpha}_0 = (1-\alpha)^{W-1} \left(c1 + c2(1 + \alpha(W-1)) \right)$

and $\alpha_m = \alpha(1-\alpha)^m \left(c1 + \alpha c2(1+m) \right)$.

The expected value of the forward predicted noise mean is

$$E[\mu f_k] = \bar{\alpha}_0 \mu_{z_{k0}} + \sum_{m=0}^{W-2} \alpha_m \mu_{z_{k-G-1-m}}. \tag{8}$$

Utilizing (7) the variance of the forward smoothing function becomes

$$\sigma^2_{\mu f_k} = E\{\mu f_k^2\} - E\{\mu f_k\}^2 \tag{9}$$

$$= \bar{\alpha}_0^2 \sigma^2_{z_{k0}} + 2 \bar{\alpha}_0 \sigma_{z_{k0}} \underline{\alpha}^T R \underline{\sigma}_{z_{k-G-1-m}} + \underline{\alpha}^T \bar{R} \underline{\alpha}$$

where $R = \begin{bmatrix} r_{W-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & r_1 \end{bmatrix}$ is a diagonal matrix and

$$\bar{R} = \begin{bmatrix} r_0 \sigma^2_{z_{k-G-1}} & r_1 \sigma_{z_{k-G-1}} \sigma_{z_{k-G-2}} & \cdots & r_{W-2} \sigma_{z_{k-G-1}} \sigma_{z_{k-G-W+1}} \\ r_1 \sigma_{z_{k-G-2}} \sigma_{z_{k-G-1}} & r_0 \sigma^2_{z_{k-G-2}} & \cdots & r_{W-3} \sigma_{z_{k-G-2}} \sigma_{z_{k-G-W+1}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{W-2} \sigma_{z_{k-G-W+1}} \sigma_{z_{k-G-1}} & r_{W-3} \sigma_{z_{k-G-W+1}} \sigma_{z_{k-G-2}} & \cdots & r_0 \sigma^2_{z_{k-G-W+1}} \end{bmatrix}$$

is a symmetric matrix. The vector $\underline{\alpha}$ where

$$\underline{\alpha}^T = [\alpha_0 \quad \alpha_1 \quad \cdots \quad \alpha_{W-2}]$$

is a vector of the forward smoothing/prediction filter weights which are a function of the filter constant α, the window size W and gap size G. The vector $\underline{\sigma}$ where

$$\underline{\sigma}^T_{z_{k\text{-}G\text{-}1\text{-}m}} = \left[\sigma_{z_{k\text{-}G\text{-}1}}\ \sigma_{z_{k\text{-}G\text{-}2}} \cdots \sigma_{z_{k\text{-}G\text{-}W+1}}\right]$$

represents the standard deviations of the samples which are mapped into the forward window which have been median processed. The correlation coefficients r_{i-j} are those of the median processed samples z_i and z_j.

The output of the recursive forward smoothing/prediction filter (7) is the sum of the filter response due to the initial state of the filter and the convolution sum of the filter weights and the median processed input. The expected value and variance of the forward noise mean estimate are given in (8) and (9) respectively.

3.0 BACKWARD SMOOTHING AND PREDICTION

Following the same procedures as in (2.0), the output of the backward two pole recursive smoothing function is

$$\underline{sb}_j = A\underline{sb}_{j+1} + \underline{b}z_j \tag{9}$$

where $j = \{k+G+W-1, \ldots, k+G+1\}$ and A and $\underline{b}$ is as defined before. The backward predicted noise mean estimate at time k is

$$\mu b_k = \underline{c}^T\ \underline{sb}_{k+G+1}. \tag{10}$$

Again solving for the output of the backward smoothing filter iteratively you get

$$\underline{sb}_j = A^{k1\text{-}j}\ \underline{1}\, z_{k1} + \sum_{\ell=k1-1}^{j} A^{\ell\text{-}j}\ \underline{b}\, z_\ell \tag{11}$$

where $k1 = k+G+W$. The filter is initialized at $j = k1$ where $\underline{sb}_{k1} = \underline{1}\, z_{k1}$.

The expected value of the backward smoothing function is

$$\mu_{\underline{sb}_j} = E\{\underline{sb}_j\}$$

(12)

$$= A^{kl-j}\ \underline{l}\, E\{z_{kl}\} + \sum_{\ell=kl-l}^{j} A^{\ell-j}\ \underline{b}\, E\{z_\ell\}$$

$$= A^{kl-j}\ \underline{l}\, \mu_{z_{kl}} + \sum_{\ell=kl-l}^{j} A^{\ell-j}\ \underline{b}\, \mu_{z_\ell}.$$

Using (10) the backward predicted noise mean estimate at time k is

$$\mu b_k = \underline{c}^T \left[A^{W-l}\ \underline{l}\, z_{kl} + \sum_{\ell=kl-l}^{k+G+l} A^{\ell-k-G-l}\ \underline{b}\, z_\ell \right]. \tag{13}$$

$$= \bar{\alpha}_0\, z_{kl} + \sum_{m=0}^{W-2} \alpha_m\, z_{k+G+l+m}.$$

The expected value of the backward predicted noise mean estimate is

$$E[\mu b_k] = \bar{\alpha}_0\, \mu_{z_{kl}} + \sum_{m=0}^{W-2} \alpha_m\, \mu_{z_{k+G+l+m}}. \tag{14}$$

The variance of the backward predicted noise mean estimate becomes

$$\sigma^2_{\mu b_k} = E\{\mu b_k^{\,2}\} - E\{\mu b_k\}^2 \tag{15}$$

$$= \bar{\alpha}_0^2\, \sigma^2_{z_{kl}} + 2\, \bar{\alpha}_0\, \sigma_{z_{kl}}\, \underline{\alpha}^T R\, \underline{\sigma}_{z_{k+G+l+m}} + \underline{\alpha}^T \bar{\bar{R}}\, \underline{\alpha}$$

where

$$\bar{\bar{R}} = \begin{bmatrix} r_0\sigma^2_{z_{k+G+1}} & r_1\sigma_{z_{k+G+1}}\sigma_{z_{k+G+2}} & \cdots & r_{W-2}\sigma_{z_{k+G+1}}\sigma_{z_{k+G+W-1}} \\ r_1\sigma_{z_{k+G+2}}\sigma_{z_{k+G+1}} & r_0\sigma^2_{z_{k+G+2}} & \cdots & r_{W-3}\sigma_{z_{k+G+2}}\sigma_{z_{k+G+W-1}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{W-2}\sigma_{z_{k+G+W-1}}\sigma_{z_{k+G+1}} & r_{W-3}\sigma_{z_{k+G+W-1}}\sigma_{z_{k+G+2}} & \cdots & r_0\sigma^2_{z_{k+G+W-1}} \end{bmatrix},$$

and $\underline{\sigma}^T_{z_{k+G+1+m}} = \left[\sigma_{z_{k+G+1}}\ \sigma_{z_{k+G+2}} \cdots \sigma_{z_{k+G-W-1}}\right]$.

The results for the backward noise mean estimate is exactly the same as those of the forward noise mean estimate except that now the samples are those mapped in from the backward window.

4.0 FINAL PREDICTED OUTPUT NOISE MEAN ESTIMATE STATISTICS

The final predicted noise mean estimate at time k is the average of the forward and backward predicted noise means given by

$$\mu_k = \frac{1}{2}\,(\mu f_k + \mu b_k). \tag{16}$$

The expected value of the final noise mean estimate is

$$E\{\mu_k\} = \frac{1}{2}\,(E\{\mu f_k\} + E\{\mu b_k\}) \tag{17}$$

where $E\{\mu f_k\}$ and $E\{\mu b_k\}$ are given in equations (8) and (14) respectively.

The variance of the final normalizer noise mean estimate is given by the sum of the variances of the forward and backward noise mean estimates and the covariance between the forward and backward noise mean estimates.

The variance of the final noise mean estimate is

$$\sigma^2_{\mu_k} = E\{\mu_k{}^2\} - E\{\mu_k\}^2 \tag{18}$$

$$= \frac{1}{4}\left(\sigma^2_{\mu f_k} + \sigma^2_{\mu b_k} + 2\,cov(\mu f_k,\mu b_k)\right)$$

and the covariance between the forward and backward predicted noise mean estimates is

$$\begin{aligned} cov(\mu f_k,\mu b_k) &= \alpha_0^2\sigma_{z_{k0}}\sigma_{z_{k1}}r_{k1-k0} + \alpha_0\sigma_{z_{k1}}\,\underline{\alpha}^T\,\hat{R}\,\underline{\sigma}_{z_{k-G-1-m}} \\ &+ \alpha_0\sigma_{z_{k0}}\,\underline{\alpha}^T\,\hat{R}\,\underline{\sigma}_{z_{k+G+1+m}} + \underline{\alpha}^T\,\hat{\hat{R}}\,\underline{\alpha} \end{aligned} \tag{19}$$

where $\hat{R} = \begin{bmatrix} r_{W+2G+1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & r_{2W+2G-1} \end{bmatrix}$ is a diagonal matrix and

$$\hat{\hat{R}} = \begin{bmatrix} r_{2G+2}\sigma_{z_{k-G-1}}\sigma_{z_{k+G+1}} & r_{2G+3}\sigma_{z_{k-G-1}}\sigma_{z_{k+G+2}} & \cdots & r_{2G+W}\sigma_{z_{k-G-1}}\sigma_{z_{k+G+W-1}} \\ r_{2G+3}\sigma_{z_{k-G-2}}\sigma_{z_{k+G+1}} & r_{2G+4}\sigma_{z_{k-G-2}}\sigma_{z_{k+G+2}} & \cdots & r_{2G+W+1}\sigma_{z_{k-G-2}}\sigma_{z_{k+G+W-1}} \\ \vdots & \vdots & \ddots & \vdots \\ r_{2G+W}\sigma_{z_{k-G-W+1}}\sigma_{z_{k+G+1}} & r_{2G+W+1}\sigma_{z_{k-G-W+1}}\sigma_{z_{k+G+2}} & \cdots & r_{2G+2W-2}\sigma_{z_{k-G-W+1}}\sigma_{z_{k+G+W-1}} \end{bmatrix}.$$

5.0 MATRIX POLYNOMIAL REDUCTION

The evaluation of the expected value and variance of the noise mean estimate requires the computation of matrix polynomials (m-fold products) which can be time consuming as the order of the polynomial becomes large and does not provide proper insight to the structure of the solution. Use of a matrix polynomial reduction technique can reduce greatly the number of computations necessary and presents the matrix polynomial in closed form.

CAYLEY-HAMILTON TECHNIQUE [2]

For a matrix polynomial $N(A)$ of degree higher than A, if $N(\lambda)$ is divided by the characteristic polynomial of A then

$$\frac{N(\lambda)}{P(\lambda)} = Q(\lambda) + \frac{R(\lambda)}{P(\lambda)} \tag{20}$$

where $R(\lambda)$ is the remainder. Multiplying equation (20) by $P(\lambda)$ you get

$$N(\lambda) = Q(\lambda)\, P(\lambda) + R(\lambda).$$

If $P(\lambda) = 0$ then $N(\lambda) = R(\lambda)$.

Let $N(A) = A^m$ so that $N(\lambda) = \lambda^m$. Then the characteristic polynomial $P(\lambda)$ of matrix A is

$$P(\lambda) = |\lambda I - A| \tag{21}$$

$$= \lambda^2 - 2\beta\lambda + \beta^2$$

where $\beta = 1 - \alpha$, I is the (2x2) identity matrix and matrix A is defined in (2).

By polynomial division as shown in equation (20) you get

$$N(\lambda) = \sum_{i=1}^{m-1} i\, \beta^{i-1}\, \lambda^{m-i-1} + \frac{m\lambda\beta^{m-1} - (m-1)\beta^m}{\lambda^2 - 2\beta^2 + \beta^2}. \tag{22}$$

The first term in equation (22) corresponds to $Q(\lambda)$ and the second term is the remainder term corresponding to $R(\lambda)/P(\lambda)$. Since $N(\lambda) = R(\lambda)$ because $P(\lambda) = 0$, the matrix polynomial A^m reduces to a closed form given by

$$A^m = R(A) = m\beta^{m-1} A - (m-1)\, \beta^m I \tag{23}$$

$$= (1-\alpha)^m \begin{bmatrix} 1 & 0 \\ m\alpha & 1 \end{bmatrix}$$

6.0 NUMBER OF INDEPENDENT SAMPLES

The effective number of independent samples in the normalizer window is required to be sufficiently large to achive the desired random error smoothing of the normalizer estimate. This value is determined from the ratio of the variance of the normalizer test cell to the variance of the normalizer noise mean estimate. The optimum achievable performance of a normalizer is its performance in stationary noise. Here performance is determined solely by the bias in the normalizer mean estimate when compared to the true mean of the test cell and the variance of the estimate. The larger the number of independent samples in the normalizer window the smaller the variance of the noise mean estimate and therefore the better the performance.

The effective number of independent samples is given by the ratio

$$W_{IID} = \frac{\sigma_{z_k}^2}{\sigma_{\mu_k}^2}. \tag{24}$$

In stationary noise $\sigma_{z_i} = \sigma_z$ and the variance of the normalizer noise mean estimate is given by

$$\sigma_\mu^2 = \frac{\sigma_z^2}{2}\left\{\bar{\alpha}_0^2(1 + r_{k1\text{-}k0}) + 2\bar{\alpha}_0 \sum_{m=0}^{W-2}\alpha_m\left(r_{W-1-m} + r_{W+2G+1+m}\right)\right\}$$
$$+ \frac{\sigma_z^2}{2}\sum_{m=0}^{W-2}\sum_{n=0}^{W-2}\alpha_m\,\alpha_n\left(r_{m\text{-}n} + r_{2G+2+m+n}\right). \tag{25}$$

7.0 NOISE MEAN ESTIMATE BIAS

The bias in stationary noise can easily be determined. The expected value of the forward noise mean estimate defined in (8) is

$$E[\mu f_k] = \tilde{\alpha}_0 \mu_z + \sum_{m=0}^{W-2} \alpha_m \mu_z$$

$$= \mu_z \left(\tilde{\alpha}_0 + \sum_{m=0}^{W-2} \alpha(1-\alpha)^m (c1 + c2(1+\mathrm{m})) \right). \qquad (26)$$

The forward predicted mean in stationary noise is given by a geometric series. Using the identities

$$\sum_{m=0}^{W-2} \beta^m = \frac{1-\beta^{W-1}}{1-\beta}$$

$$\sum_{m=0}^{W-2} m\beta^m = \frac{\beta^{W-1}}{(1-\beta)^2}\left(1 - \beta^{W-2} - (W-2)\beta^{W-2} + (W-2)\beta^{W-1}\right)$$

equation (26) reduces to

$$E[\mu f_k] = \mu_z (c_1 + c_2).$$

But $c_2 = 1 - c_1 \equiv c_1 + c_2 = 1$. Therefore $E[\mu f_k] = \mu_z$. The bias in the normalizer mean estimate in stationary noise is zero. Similar procedure shows the same is true for the backward prediction.

References:

[1] Dr. C. J. Wenk - "Normalizer and Noise Mean Estimator Specification for Inclusion is the SALFAS PIDS", A&T Engineering Technology Center, Tech Memo No. ETC: 92:08-61, 23 September 1992.

[2] Dr. C.J. Wenk - "AN Analytic Design and Performance Evaluation Methodology for Active Sonar Normalizers", A&T Engineering Technology Center, Report No. P-4578-5-92, 15 March 92.

[3] P.M. Derusso, R.J. Roy, C.M. Close - "State Variables for Engineers", John Wiley & Sons, Inc.

[4] R.A. Gabel, R.A Roberts - "Signals and Linear Systems", second edition, John Wiley & Sons, Inc.

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